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## DESIGN OF AUTOPILOTS FOR BANK-TO-TURN MISSILES

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### Abstract

A method for the design of autopilots for bank-to-turn missiles is described. Based on linear-gaussian-quadratic control theory, the method makes use of the "adiabatic approximation" to deal with cross-coupling between the pitch and yaw axes of the missile proportional to roll rate. A software package has been developed to perform all the required design calculations, performance evaluation, and robustness assessment. This software has been used for the design of autopilots for several prospective airframes with results that achieve desired performance objectives. The control architecture is unlike that which a classical design would achieve, but performance of the two designs are comparable.

### Introduction

Bank-to-turn missiles develop their steering force by banking to orient the aerodynamic normal force in the direction commanded by the guidance law. The design of autopilots for such missiles is a challenge because the order of the dynamic model is relatively high and all the axes are coupled. The coupling of the pitch and yaw axes, moreover, due to coriolis and gyroscopic effects, is significant and nonlinear.

Over the past several years a methodology for the design of autopilots for bank-to-turn missiles has been developed by investigators at the Kearfott Guidance and Navigation Corporation, under partial sponsorship of the U.S. Air Force Armament Laboratory. This methodology is based on the approximate decoupling of the roll axis dynamics from the dynamics of the pitch and yaw axes, for which the couplings are retained in the design model. This design model leads to two controllers: one for the roll axis, which is quite conventional in structure, and one for the coupled pitch/yaw axes. The latter is designed with the aid of the "adiabatic approximation" principle by which the roll rate is treated as a slowly time-varying parameter. The unmeasured states are es-

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timated by means of an observer having the configuration of a reduced-order extended Kalman filter.

### Dynamic Model of Missile

The objective in the design of autopilots for bank-to-turn missiles is to achieve maneuverability comparable to that which can be achieved with skid-to-turn missiles. To achieve this maneuverability entails high roll rates concurrent with changes in missile attitude. Thus all the axes of the missile are strongly coupled. A control system design based on a model that omits the inter-axis coupling is liable to perform inadequately, as our early experience has confirmed. Consequently it is important to include the cross-axis coupling in the design model and to account for it explicitly in the control system design. Thus the design model adopted for the the missile dynamics is the following:

Pitch Axis:

$$\begin{aligned}\dot{\epsilon}_s &= \frac{Z_\alpha}{V}(\epsilon_s + a_{sc}) + Z_\alpha q + Z_{\delta q} \dot{\delta}_q - [Z_\alpha(\epsilon_y + a_{yc}) - Z_\alpha Y_{\delta r} \delta_r] \frac{p}{Y_\beta} \\ \dot{q} &= \frac{M_\alpha}{Z_\alpha}(\epsilon_s + a_{sc}) + M_q q + (M_{\delta q} - \frac{Z_{\delta q} M_\alpha}{Z_\alpha}) \delta_q - (\frac{I_{zz} - I_{xx}}{I_{yy}} r) p \\ \ddot{\delta}_q &= \omega^2(u_{\delta q} - \delta_q) - 2\zeta\omega\dot{\delta}_q\end{aligned}\quad (1)$$

Yaw Axis:

$$\begin{aligned}\dot{\epsilon}_y &= \frac{Y_\beta}{V}(\epsilon_y + a_{yc}) - Y_\beta r + Y_{\delta r} \dot{\delta}_r - [Y_\beta(\epsilon_s + a_{sc}) - Y_\beta Z_{\delta q} \delta_q] \frac{p}{Z_\alpha} \\ \dot{r} &= \frac{N_\beta}{Y_\beta}(\epsilon_y + a_{yc}) + N_r r + (N_{\delta r} - \frac{Y_{\delta r} N_\beta}{Y_\beta}) \delta_r - (\frac{I_{zz} - I_{yy}}{I_{xx}} q) p \\ \ddot{\delta}_r &= \omega^2(u_{\delta r} - \delta_r) - 2\zeta\omega\dot{\delta}_r\end{aligned}\quad (2)$$

Roll Axis:

$$\begin{aligned}\dot{\epsilon}_\phi &= p \\ \dot{p} &= L_p p + L_{\delta p} u_{\delta p} + L_\beta \beta\end{aligned}\quad (3)$$

The variables in the above equations are defined as follows:

$p$	roll rate
$q$	pitch rate
$r$	yaw rate
$\alpha$	angle of attack
$\beta$	angle of sideslip
$e_\phi$	bank angle error
$a_{x_c} = a_x - e_x$	commanded acceleration in vertical direction
$a_{y_c} = a_y - e_y$	commanded acceleration in lateral direction
$\delta_p$	effective roll control surface deflection
$\delta_q$	effective pitch control surface deflection
$\delta_r$	effective yaw control surface deflection

The aerodynamic coefficients are denoted by  $Z, Y, L, M, N$  with appropriate subscripts, in accordance with customary usage;  $V$  is the missile speed. It is noted that the "effective" control surface deflections are derived as linear combinations of the actual control surface deflections which depend on the specific missile geometry.

There are two forms of cross-coupling between the pitch and yaw axes: gyroscopic crosscoupling as represented by the terms containing the missile inertia components  $I_{zz}, I_{yy}$  and  $I_{xx}$ , and coriolis crosscoupling represented by the terms proportional to the roll rate. In practical cases the gyroscopic crosscouplings may be negligible, but the coriolis terms may account for a significant fraction of the total moments at high roll rates and cannot be neglected.

### Autopilot Design

The design approach that was adopted in this investigation is based on the separation principle: first a control law is designed as if all the states are measured and then an observer is provided to estimate the states that cannot be measured directly. In this application, it is assumed that the instrumentation of the missile provides direct measurement of the body rates ( $p, q$ , and  $r$ ) by means of rate gyros, and the components of acceleration along the body axes, by means of accelerometers. The instruments are assumed to be of navigation quality and hence, for design purposes, essentially error free. This means that it is not necessary to estimate these variables. Thus it is necessary only to estimate the states of the actuators. To provide integral control, however, it is assumed that the commanded accelerations must also be estimated from the measured acceleration errors, even the acceleration commands are directly available. (The use of this technique to provide integral control is discussed in Reference[1].) The design of the appropriate reduced-order observer is a relatively straightforward process the details of which are omitted in this paper; they are included in Reference [2].

The full-state feedback control law design uses the "adiabatic approximation" [3] which is based on the assumption that the roll-rate can be regarded

as a constant in the design of the feedback law for the pitch/yaw channel. As a consequence of the adiabatic approximation, the control gains can be scheduled as functions of the roll rate.

The structure that results from combining the gain-scheduled full-state feedback for the pitch/yaw channel with the observer is shown in Figure 1. Since the aerodynamic derivatives are also functions of the dynamic pressure which typically vary over the range of an order of magnitude, each of the control gains is actually a function of two variables: roll rate and dynamic pressure. The dependence on the latter is omitted in Figure 1 for simplicity.

The design of the control for the roll channel is relatively straightforward. The bank angle error is computed from the commanded vertical and lateral accelerations:

$$\epsilon_\phi = \tan^{-1}(a_{yc}/a_{sc}) \quad (4)$$

where  $a_{yc}$  and  $a_{sc}$  are the lateral and vertical acceleration commands obtained from the missile's guidance system. The control gains for the roll channel can be obtained by standard methods which do not require any explanation here.

The 20 gains for the pitch/yaw channel are computed to minimize a quadratic function

$$V = \int_t^\infty (C_1^2 \epsilon_x^2 + C_2^2 \epsilon_y^2 + C_3^2 \beta^2 + C_4^2 \dot{\delta}_q^2 + C_5^2 \dot{\delta}_r^2 + C_6^2 u_{\delta_q}^2 + C_7^2 u_{\delta_r}^2) d\tau \quad (5)$$

The weighting coefficients  $C_i$  in (5) are selected to achieve a suitable compromise between performance and control effort. Note that (5) has a weighting on the sideslip angle but no weighting on the angle of attack. This is because a bank-to-turn missile obtains its maneuvering acceleration primarily through its angle of attack. The sideslip, on the other hand, is generally required to be small for good performance. In some cases (e.g., air-breathing missiles) an excessive sideslip can extinguish the engine.

The computation is performed with the aid of a software package BTTPACK [4] which solves the algebraic matrix Riccati equation for each of a number of combinations of dynamic pressure and roll rates spanning the range of operation of the missile. A two-dimensional surface is fit to the results by one of the BTTPACK programs that uses rule-based heuristics. Also included in BTTPACK are procedures for evaluating the robustness of the design by computing the singular-value plots of the loop transmission. The calculations that can be performed by BTTPACK are shown in the flowchart of Figure 2.

To perform an autopilot design, the user provides the BTTPACK program with the aerodynamic data and makes an initial selection of the weighting factors in (5) and the noise parameters for the reduced-order Kalman filter. Using this data the program automatically performs all the autopilot design calculations. The program user can then evaluate the effectiveness of the design by

performing six-degree-of-freedom simulations (M6DOF) or robustness assessments (SINVAL). If, in the judgment of the user, the performance meets the requirements, the design is complete. If not, the user changes the weighting factors and repeats the process until a satisfactory design is achieved.

### Performance Evaluation

The effectiveness of this design methodology has been demonstrated on autopilot designs for a number of conceptual tactical missiles of differing aerodynamic characteristics. In all cases the methodology provided a satisfactory design within several weeks of the time the design was initiated.

Time-histories of several of the variables in a six-degree-of-freedom closed-loop guidance system simulation for a typical design [2] are shown in Figures 3 and 4. The miss distances for the two cases were 2.1 ft and 6.7 ft respectively. Robustness of the design was also investigated for the design at a zero roll rate (the steady-state condition). The guaranteed margins were found to be [-10.8 dB, 8.7 dB] and  $\pm 41.6$  degrees, which are acceptable in this application.

An extensive comparison was made for another missile of the autopilot design obtained with this methodology with a design obtained by application of classical, frequency domain techniques by highly experienced designers. Not surprisingly, both designs performed adequately, notwithstanding substantial differences in the architecture of the two designs[5]. The architectural differences, in fact, made it all but impossible to make some types of performance evaluations that are typically conducted for missile autopilots. For example, in the classical design it is customary to assess robustness by opening feedback loops that are not considered in the robustness assessment of the state-space design. Under some circumstances it could turn out that opening of these loops would render the latter design unstable. But it is doubtful whether the offending loop could open under in a real missile with a digitally-implemented autopilot.

### Conclusions

This investigation has demonstrated that state-space methods can be effectively employed to design controllers for complex dynamic processes with severe performance requirements: bank-to-turn missiles. With the aid of appropriate software, moreover, it is possible for the design to be accomplished quickly, by relatively inexperienced design engineers. The resulting performance rivals that of the design produced by engineers with many years of design experience.

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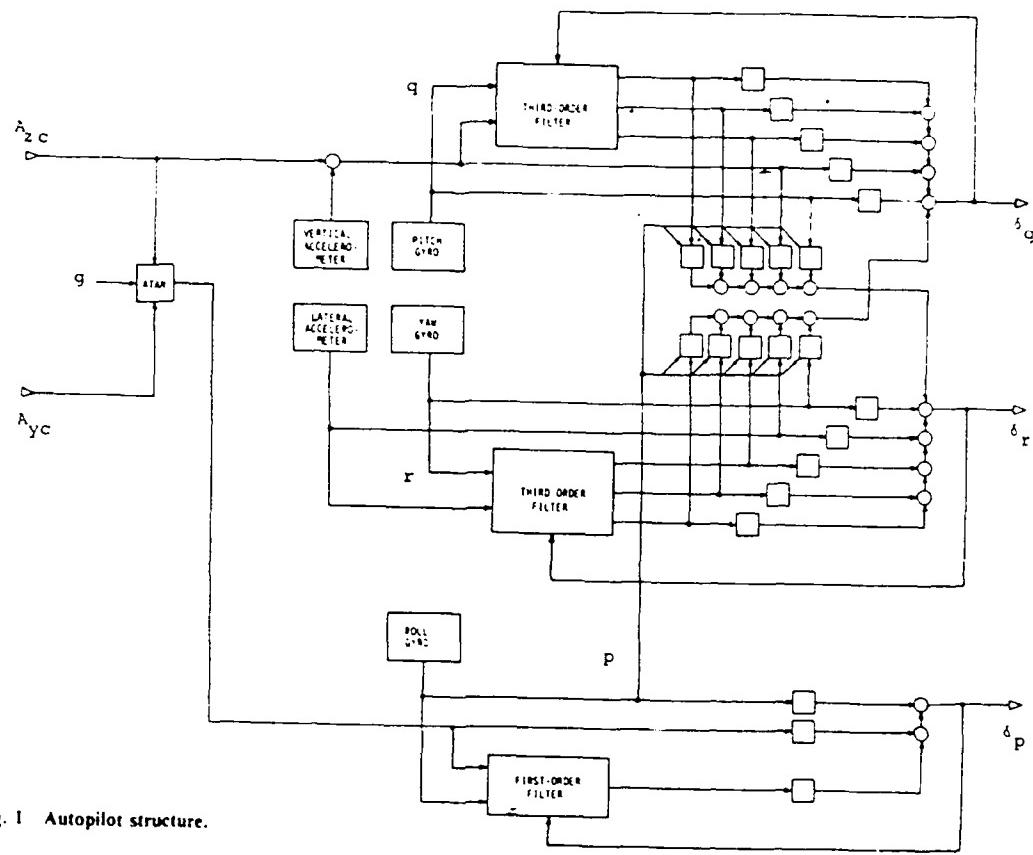


Fig. 1 Autopilot structure.

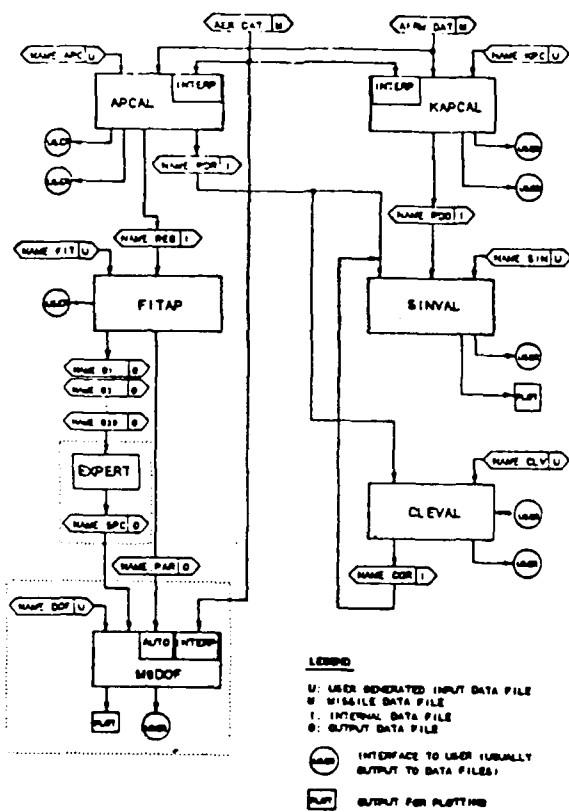


Figure 2: BTTPACK Data Flow

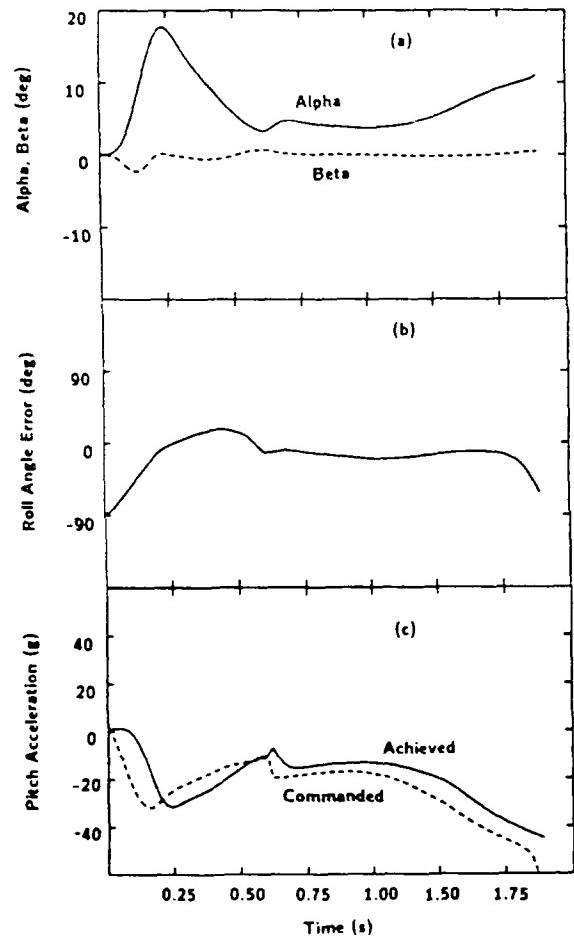


Fig. 3 Intercept time histories at design altitude of 10,000 ft.

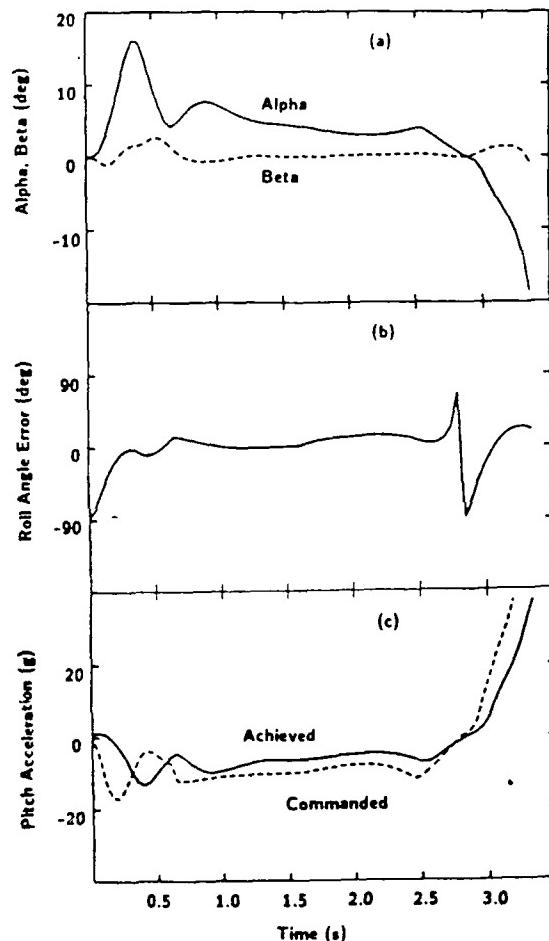


Fig. 4 Interception time histories at altitude of 40,000 ft.